



166

Total No. of Questions - 24
Total No. of Printed Pages - 3

Regd. No.

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Part - III
MATHEMATICS, Paper - I (A)
(Algebra and Trigonometry)
(English Version)

Time : 3 hours

Max. Marks : 75

Note : This question paper consists of **three** sections A, B and C.

SECTION A

I. Very short answer type questions.

10 x 2 = 20

- Answer **all** questions.
- Each question carries **two** marks.

- If f and g are real valued functions defined by $f(x) = 2x - 1$ and $g(x) = x^2$, then find :
 - $(fg)(x)$
 - $(f+g+2)(x)$
- Find the domain and range of the function $f(x) = \frac{x}{2-3x}$.
- Let $\bar{a} = 2i + 4j - 5k$, $\bar{b} = i + j + k$, $\bar{c} = j + 2k$. Find the unit vector in the opposite direction of $(a + b + c)$.
- $OABC$ is a parallelogram. If $OA = a$ and $OC = c$, find the vector equation of the side BC .
- Find the radius of the sphere whose equation is $r^2 = 2r \cdot (4i - 2j + 2k)$.
- $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$. Prove that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$.

7. Find the maximum and minimum values of $\cos\left(x + \frac{\pi}{3}\right) + 2\sqrt{2} \sin\left(x + \frac{\pi}{3}\right) - 3$.
8. If $\sin hx = \frac{1}{2}$, find the value of $\cosh 2x + \sinh 2x$.
9. If $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$, then show that $\triangle ABC$ is an equilateral triangle.
10. If $z_1 = -1, z_2 = i$, then find the value of $\operatorname{Arg}\left(\frac{z_1}{z_2}\right)$.

SECTION B

II. Short answer type questions.

5 x 4 = 20

- i) Attempt **any five** questions.
- ii) Each question carries **four** marks.
11. If a, b, c are linearly independent vectors, then show that $a - 2b + 3c, -2a + 3b - 4c, -b + 2c$ are linearly dependent.
12. If $a = 2i + j - k, b = -i + 2j - 4k$ and $c = i + j + k$ then find $(a \times b) \cdot (b \times c)$.
13. If A is not an integral multiple of π , prove that $\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16 \sin A}$.
14. Find the values of x in $(-\pi, \pi)$ satisfying the equation $8^{1 + \cos x + \cos^2 x + \dots \dots \dots \infty} = 4^3$.
15. Solve the equation $3 \sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4 \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2 \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$.
16. Prove that $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \frac{S^2}{\Delta}$.
17. Show that $\frac{\sin 6\theta}{\sin \theta} = 32 \cos^5 \theta - 32 \cos^3 \theta + 6 \cos \theta$ when $\sin \theta \neq 0$.

SECTION C

III. Long answer type questions.

5 x 7 = 35

- i) Attempt **any five** questions.
- ii) Each question carries **seven** marks.

18. If $f : A \rightarrow B$, $g : B \rightarrow C$ be bijections, then show that $g \circ f : A \rightarrow C$ is a bijection.

19. Using mathematical induction, prove that

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots \text{ upto } n \text{ terms} = \frac{n(n+1)^2(n+2)}{12}.$$

20. Find the shortest distance between the skew lines

$$r = (6i + 2j + 2k) + t(i - 2j + 2k) \text{ and } r = (-4i - k) + s(3i - 2j - 2k).$$

21. If $A + B + C = 180^\circ$, then prove that

$$\frac{\sin A + \sin B + \sin C}{\sin A + \sin B - \sin C} = \cot \frac{A}{2} \cot \frac{B}{2}.$$

22. If $r_1 = 2$, $r_2 = 3$, $r_3 = 6$ and $r = 1$, prove that $a = 3$, $b = 4$ and $c = 5$.

23. From a point B on the level ground away from the foot of the hill AD , the top of the hill makes an angle of elevation α . From the point B , the point C is reached by moving a distance ' d ' along a slant / slope which makes an angle γ with the horizontal. If β is the angle of elevation of the top of the hill from C , find the height of the hill.

24. If n is an integer, then show that $(1+i)^{2n} + (1-i)^{2n} = 2^{n+1} \cos \frac{n\pi}{2}$.